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The observer's own remarks are :—

(6) Clearly past conjunction 4^m after.

(7) A good observation of mid-transit of Shadow of IV.

(8) Sat. IV. appeared as a tolerably round, very dark, almost black spot.

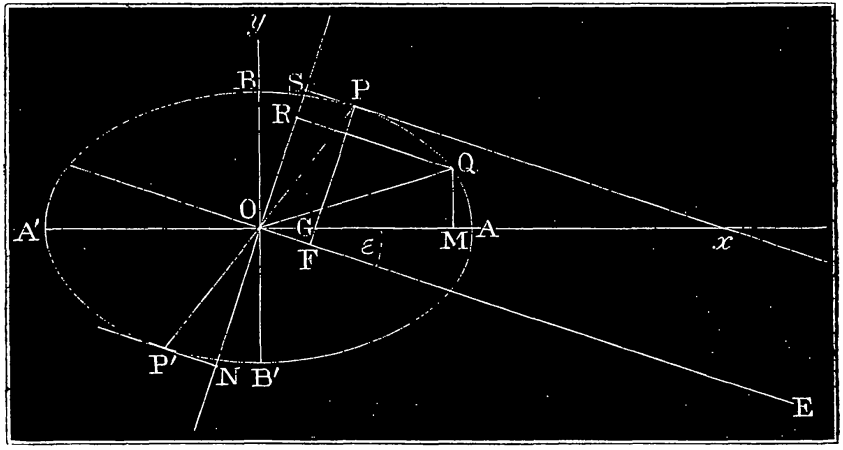
In the case of (7) and (8) the predicted times are inferred from the *Nautical Almanac* for 1890.

Murston Rectory, Sittingbourne :
1893 *November* 8.

Method of Finding the Latitudes of Saturn's Belts.

By the Rev. A. Freeman, M.A.

Numerous drawings of *Saturn* having been submitted to my inspection in the past two years, as drawn by several observers, it seems fit to show how they may be used for the purpose of determining the kronocentric latitudes of the edges of belts of the planet by measurement of the distance, taken along the central meridian, from the centre of the planet's disc to the point where the edge of any belt cuts that meridian. By this meridian is meant the line joining the apparent poles of the disc.



In the figure let the ellipse ABA^1B^1 be a section of *Saturn* by a plane passing through the Earth and the least axis of *Saturn*.

Let OE be the direction of the Earth, SPx a tangent parallel to OE, SON a straight line perpendicular to OE.

Then the apparent disc of *Saturn* is projected on the plane through SON perpendicular to OE, by straight lines in the direction of EO.

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The apparent poles S, N are the projections of P and P^1.

PGF is normal to the elliptic section at P, Q is any point in this section which is projected in R upon SN, the central meridian of the disc of *Saturn*.

Let

$$\angle\ \mathrm{POA} = \theta,\ \mathrm{OP} = r,$$

then

$$\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2} = \frac{1}{r^2}$$

if a and b are the semi-axes of *Saturn*.

Let ϵ = EOA be the elevation of the Earth north of *Saturn's* equator,

$$\mathrm{PF} = r\,.\,\sin(\theta+\epsilon) = \frac{b\,.\,\sin(\theta+\epsilon)}{\sqrt{1-e^2\cos^2\theta}},$$

where

$$b^2 = a^2(1-e^2).$$

Again, let

$$\mathrm{QOA} = \phi,\ \mathrm{OQ} = r',$$

then

$$r' = \frac{b}{\sqrt{1-e^2\cos^2\phi}},$$

and, if OM, QM be the co-ordinates of Q,

$$\mathrm{OM} = \frac{b\cos\phi}{\sqrt{1-e^2\cos^2\phi}},\quad \mathrm{QM} = \frac{b\sin\phi}{\sqrt{1-e^2\cos^2\phi}}.$$

Now, QR being perpendicular to OS and parallel to EO,

$$\mathrm{OR} = \mathrm{OM}\,.\,\sin\epsilon + \mathrm{QM}\,.\,\cos\epsilon$$
$$= \frac{b\sin(\phi+\epsilon)}{\sqrt{1-e^2\cos^2\phi}}.$$

To find the relation between θ and ϵ, by consideration of the normal at P being perpendicular to OE, we have

$$\tan\epsilon = \frac{b^2\cos\theta}{a^2\sin\theta},\ \text{or}\ \cot\theta = \frac{\tan\epsilon}{1-e^2}.$$

Hence it will be possible to substitute for θ in the expression for PF, so that we shall have

$$\mathrm{PF} = a\sqrt{1-\cos^2\epsilon},$$

after reductions.

Thus, finally,

$$\frac{\mathrm{OR}}{\mathrm{PF}} = \frac{b\,.\,\sin(\phi+\epsilon)}{a\sqrt{1-e^2\cos^2\phi}\,.\,\sqrt{1-e^2\cos^2\epsilon}}\quad .\ .\ .\ .\ .\ .\ .\quad (\mathrm{A})$$

Now, PF is the apparent polar semi-axis of *Saturn's* disc, and OR is the distance along the central meridian to a point on it which is the projection of Q, whose kronocentric latitude is ϕ.

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In 1893 the elevation of the Earth north of *Saturn's* equator ranged between 8°·802 on February 15 and 6°·093 on May 31; and it could be found from Marth's Ephemeris that the Earth's elevation was

$$\left.\begin{array}{llrr} {}^{\circ} & & \text{d} & \text{h} \\ 8\cdot5 & \text{on Feb.} & 26 & 18\cdot5 \\ 7\cdot5 & \text{on Apr.} & 2 & 6\cdot0 \\ 6\cdot5 & \text{on July} & 5 & 21\cdot0 \end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad \text{(B)}$$

In the expression (A) ϵ is positive, south latitudes answer to positive values and north latitudes to negative values of ϕ, OR being negative when R falls between O and N.

The following table, calculated by myself, gives the values of the expression (A) between south latitude 45° and north latitude 60° at intervals of 7°·5 and corresponding to the three assumed elevations (B). It will be easy by simple formulæ of interpolation to obtain from it values corresponding to the same latitudes, but to the different elevations 6°, 7°, 8°, 9°, and this will suffice for the measurement of all drawings of *Saturn* made in 1893.

We suppose

$$\frac{b}{a} = \frac{15''\cdot4}{17''\cdot3},$$

as given by Professor Oudemans (*Monthly Notices*, lii., 3), whence

$$e = 0\cdot455622$$

$$\log\left(\frac{b}{a}\right) = \bar{1}\cdot9494746, \qquad \log e = \bar{1}\cdot6586046.$$

The denominator was adapted to logarithms by assuming

$$e \cos \phi = \cos \phi_1, \text{ and } e \cos \epsilon = \cos \epsilon_1$$

Table of the Values of the Expression A.

	Lat. ϕ	$\epsilon = 8^\circ\cdot5$	$\epsilon = 7^\circ\cdot5$	$\epsilon = 6^\circ\,5$
S.	45°	+0·84671	+0·83622	+0·82530
	37·5	0·76931	0·75675	0·74379
	30	0·67555	0·66104	0·64626
	22·5	0·56617	0·55003	0·53357
	15	0·44274	0·42517	0·40741
	7·5	0·30807	0·28950	0·27071
	0	0·16557	+0·14631	+0·12696
N.	7·5	+0·01951	0·00000	−0·01954
	15	−0·12569	−0·14505	0·16431
	22·5	0·26594	0·28471	0·30336
	30	0·39773	0·41558	0·43323
	37·5	0·51849	0·53510	0·55146
	45	0·62653	0·64166	0·65648
	52·5	0·72097	0·73435	0·74746
	60	−0·80145	−0·81297	−0·82418

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I have extended this table to the elevations corresponding first of all to 7° and 8°, and then to the elevations, 6° and 9°, by formulæ of interpolation based on the method of *Boole's Finite Differences*, chap. iii. art. 4 of the third edition, assuming that if u_0, u_1, u_2, u_3, u_4, u_5, u_6 be the values of the expression (A) corresponding to a constant latitude ϕ, but to elevations differing by half a degree,

$$8u_2 = 3u_1 + 6u_3 - u_5 \text{ and } 8u_4 = 3u_5 + 6u_3 - u_1,$$

and finally that, third differences assumed zero,

$$u_0 = u_3 + 3(u_1 - u_2), \text{ and } u_6 = u_3 - 3(u_4 - u_5).$$

The rest of the table now follows :—

Lat. ϕ		$\epsilon=9°$	$\epsilon=8°$	$\epsilon=7°$	$\epsilon=6°$
S.	45°	+0·85179	+0·84152	+0·83081	+0·81969
	37·5	0·77553	0·76305	0·75309	0·72945
	30	0·68273	0·66832	0·65368	0·63878
	22·5	0·57512	0·55814	0·54184	0·52522
	15	0·45145	0·43398	0·41631	0·39847
	7·5	0·31728	0·29881	0·28013	0·26124
	0	0·17520	0·15594	0·13665	+0·11724
N.	7·5	+0·02925	+0·00976	−0·00975	−0·02937
	15	−0·11769	−0·13571	−0·15456	−0·17430
	22·5	0·25631	0·27534	0·29405	0·31264
	30	0·38893	0·40668	0·42418	0·44273
	37·5	0·51008	0·52683	0·54331	0·55955
	45	0·61511	0·63538	0·64911	0·66377
	52·5	0·71419	0·72769	0·74094	0·75391
	60	−0·79557	−0·80725	−0·81861	−0·82968

Murston Rectory, Sittingbourne :
1893 *November* 6.

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Observations of Phenomena of Jupiter's Satellites at Windsor, New South Wales, in the Year 1892.

By John Tebbutt.

Day of Obs.	Satellite.	Phenomenon.	Phase.	Aperture of Telescope.	Mag. Power.	Greenwich Mean Time of Observation.	Mean Time of *Nautical Almanac.*
1892.						h m s	h m s
Oct. 29	I.	Tr. Ingr.	Ext. contact	8-inch	230	22 10 23	
29	I.	,,	Bisection	,,	,,	22 12 23	22 10
29	I.	,,	Int. contact	,,	,,	22 13 48	
30	III.	,,	Ext. contact	,,	300	0 7 19	
30	III.	,,	Bisection	,,	,,	0 14 53	0 9
30	III.	,,	Int. contact	,,	,,	0 21 47	
30	I.	Tr. Egr.	Int. contact	,,	,,	0 18 22	
30	I.	,,	Bisection	,,	,,	0 20 7	0 22
30	I.	,,	Ext. contact	,,	,,	0 22 17	
30	III.	,,	Int. contact	,,	,,	1 49 32	2 10
30	II.	Ecl. R.	First seen	,,	140	1 58 13	1 56 5
30	I.	,,	First seen	,,	170	22 3 28	22 3 27
Nov. 14	II.	Tr. Ingr.	Ext. contact	,,	,,	21 48 12	
14	II.	,,	Bisection	,,	,,	21 50 22	21 54
14	II.	,,	Int. contact	,,	,,	21 52 41	
14	I.	Tr. Egr.	Int. contact	,,	,,	22 14 28	
14	I.	,,	Bisection	,,	,,	22 17 22	22 19
14	I.	,,	Ext. contact	,,	,,	22 19 37	
16	III.	Occ. R.	Bisection	,,	,,	22 25 9	22 35
16	III.	,,	Last contact	,,	,,	22 27 24	